Computational Mathematics

Numerical Integration

**General Quadrature Formula for Equidistance:**

Let

Let f(x) be given for certain equidistance values of

Suppose are the points corresponding to the arguments:

Limit: when

After completing the integration we get:

This is the general quadrature formula.

If we put n=1 in (1) we get: Trapezoidal rule

If we put n=2 in (1) we get: Simpson’s One-Third rule

If we put n=3 in (1) we get: Simpson’s Three-Eight’s rule

If we put n=4 in (1) we get: Weddle’s rule

**Trapezoidal rule:**

Putting n=1 in the formula (1) and neglecting the **second and higher order** differences, we get:

Similarly for n=2,3,…………..

………………………………………………

………………………………………………

Adding these n integrals, we get:

**Simpson’s One-Third rule:**

Putting n=2 in the equation (1) and neglecting the **third and higher order** differences, we get:

Similarly for n=4,6,……………..

…………………………………………………

…………………………………………………

Adding all these integrals, we get:

This formula can be used only when the number of subdivisions of the interval is even.

**Simpson’s Three-Eight’s rule (3/8 rule):**

Putting n=3 in the formula (1) and neglecting the **four and higher order** differences, we get:

Similarly for n=6,9,………………

………………………………………………………….

………………………………………………………….

Adding all these integrals, we get:

This formula is used when the number of subdivisions of the interval is a multiple of 3.

**Weddle’s rule:**

Putting n=6 in (1) and neglecting the difference of orders **higher than six**, we get:

This formula requires at least **seven** consecutive values of the function.

**Ex-1:** Find by using (i) rule, (ii) rule, (iii) Trapezoidal rule, (iv) Weddle’s rule.

**Solution:** we have here: a=0; b=1;

We shall divide the interval into six equal parts. So, n=6.

Here,

Now the value of for each point of sub division are given below:

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By Simpson’s 1/3 rule, we get:

By Simpson’s 3/8 rule:

By Trapezoidal rule:

By Weddle’s rule:

* Rule and rule give more accurate result than Trapezoidal and Weddle’s rules.

**Ex-2:** Evaluate by using Simpson’s (i) rule, (ii) rule.

**Solution:** we have here: a=0; b=;

We shall divide the interval into six equal parts. So, n=6.

Here,

Now the value of for each point of sub division are given below:

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By Simpson’s 1/3 rule, we get:

By Simpson’s 3/8 rule:

**Ex-3:** Calculate by using Simpson’s rule.

**Solution:** we have here: a=0.5; b=0.7;

We shall divide the interval into 4 equal parts. So, n=4.

Here,

Now the value of for each point of sub division are given below:

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By Simpson’s 1/3 rule, we get:

**Ex-4:** Calculate the value of by using (i) Trapezoidal rule, (ii) rule, (iii) rule, (iv) Weddle’s rule.

**Solution:** we have here: a=4; b=5.2;

We shall divide the interval into 6 equal parts. So, n=6.

Here,

Now the value of for each point of sub division are given below:

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By Trapezoidal rule:

By Simpson’s 1/3 rule, we get:

By Simpson’s 3/8 rule:

By Weddle’s rule:

**Ex-5:** Compute the value by Simpson’s 1/3 rule and compare with the exact value.

**Solution:**

we have here: a=1.2; b=1.6;

We shall divide the interval into 8 equal parts. So, n=8.

Here,

* 1/3 rule = 0.8477
* Exact value = 0.8477

So, NO ERROR.